PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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Detection of Large-Scale Ocean Circulation and Tides

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Detection of large-scale ocean circulation and tides

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As emphasized recently by Munk & Wunsch, the traditional methods of monitoring the ocean circulation give data too hopelessly aliased in space and time to permit a proper assessment of basin-wide dynamics and heat flux on climatic timescales. The prospect of nearly continuous recording of ocean-surface topography by satellite altimetry with suitable supporting measurements might make such assessments possible. The associated identification of the geocentric oceanic tidal signal in the data would be an additional bonus. The few weeks of altimetry recorded by Seasat gave a glimpse of the possibilities, but also clarified the areas where better precision and knowledge are needed. Further experience will be gained from currently projected multi-purpose satellites carrying altimeters, but serious knowledge of ocean circulation will result only from missions that are entirely dedicated to the precise measurement of ocean topography.

1. Introduction

Measuring and trying to understand the dynamics of the general circulation of the oceans has always been the central problem of physical oceanography. The more that detailed measurements are made, the more complicated the dynamics appears to be, with mesoscale eddies dominating the picture of what was once thought of as steady basin-wide gyre systems. The small scales of this relatively fine structure, of order 100 days and 100 km, now make the conventional methods of monitoring ocean circulation, by means of ships, tracked floats, and arrays of moored instruments, seem rather inadequate, except for localized experiments. This situation is emphasized, colourfully, by Munk & Wunsch (1982), who predict a new generation of ocean-monitoring experiments in the last decade of this century, employing the combined techniques of acoustic tomography and radar altimetry from satellites. The present paper is concerned only with the second of these techniques.

Estimating ocean circulation by means of radar altimetry involves measuring the elevation of the smoothed ocean surface relative to a fixed equipotential surface to 0.1 m precision or better. The anomaly sought is usually less than the tidal variation of the surface, so the tidal elevation must first be removed from the direct measurements. This is particularly important for the solar tides, which tend to have long-period aliases in the altimetry signal. Many accounts of the subject ignore the tides, assuming that they can be removed by means of a computer model. The best tidal models available (Parke & Hendershott 1980; Parke 1982; Schwiderski 1980) are probably capable of giving 0.1 m precision in most oceanic regions, but this is not yet proven and there is a need to evaluate the tide directly from the altimetric measurements, especially in the extensive areas where they have not been measured previously by other means. Extracting information about the tides can therefore be regarded as an important by-product of the process of extracting the ocean circulation.

2. Definitions and theory

The fundamentals of satellite altimetry have been described by many authors, for example Wunsch & Gaposchkin (1980), whose notation I shall largely use. Here, I outline only those concepts essential as a basis for the subsequent discussion.

Let $N(\theta, \lambda)$ be the height of the gravitational equipotential surface known as the 'geoid' above a standard ellipsoid of reference, where θ, λ denotes geographical latitude and longitude coordinates. In older books on geodesy and even in fairly recent papers on satellite theory, the term geoid is used synonymously with the sea surface, but such an approximation is tolerable only when precision of no better than 1 or 2 m is assumed. In the present context, the sea surface differs essentially from the geoid by the quantity $\zeta(\theta, \lambda, t)$ which appears as an important variable in ocean dynamic theory. The *mean* value of ζ over the oceans is supposed to be zero by definition. If S is the instantaneous height of the satellite above the wave-smoothed sea surface as measured by the altimeter's radar pulse (with atmospheric and ionospheric corrections), and R is its height above the reference ellipsoid as computed from gravitational theory and tracking data, then the desired sea surface height anomaly is given by

$$\zeta(\theta, \lambda, t) = R - S - N. \tag{1}$$

(a) Circulation

Most of the currents in the ocean are in geostrophic balance with the local pressure gradients. Exceptions are in the Ekman boundary layer beneath strong winds and in the neighbourhood of strong jets. Allowances can be made for non-geostrophic forces in altimetry calculations, given supplementary data such as the wind-stress; indeed, monitoring of the wind-stress is considered to be an essential adjunct to future altimetric exercises intended for circulation studies. I shall assume geostrophy here, to demonstrate the essential principles. Tidal accelerations are of course assumed to be eliminated.

In a field of spatially variable density $\rho(x, y, z)$, where x, y, z are a right-handed set of local Cartesian coordinates with z vertically upwards, elementary principles show that the velocity v in the (arbitrary) horizontal direction of the y axis is related to the density gradient in the x direction by the 'thermal wind equation'

$$\frac{\partial(\rho v)}{\partial z} = \frac{g}{f}\frac{\partial \rho}{\partial x}, \tag{2}$$

where $f = 2 \sin \theta \times$ (Earth rotation frequency) is the Coriolis requency and g is gravitational acceleration. The problem of deducing the velocity field $v(x, y, z)$ by integrating (2) with respect to z from a supposed equipotential 'level of no motion' and the known hydrographic data for ρ has exercised oceanographers for decades. Various deep levels of no motion have been suggested and disputed, and some deny the reality of the concept altogether. Most realistically, Wunsch (1978, 1981) determines the unknown velocity field at an arbitrary reference level by solving an inverse problem posed by various continuity equations and horizontal boundary conditions. However, this method has to rely on the effective consistency of many hydrographic sections taken at different times and places.

Specifying the surface topography $\zeta(x, y)$ by altimetry enables the geostrophic velocity field at all depths normal to a satellite track to be integrated unambiguously from the surface

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

downwards. If $z = 0$ is taken at the level of the geoid, then the pressure at a depth d below the geoid is

$$p(-d) = \rho(0)g\zeta + p_0 + g\int_{-d}^{0} \rho(z)\,\mathrm{d}z, \tag{3}$$

where $\rho(z)$ has been written for $\rho(x, y, z)$ and p_0 is the atmospheric pressure at the surface. The geostrophic assumption then gives

$$v(x, y, z) = \{f\rho(0)\}^{-1}\left\{g\rho(0)\,\frac{\partial\zeta}{\partial x} + \frac{\partial p_0}{\partial x} + g\int_{-d}^{0} \frac{\partial\rho(z)}{\partial x}\,\mathrm{d}z\right\}. \tag{4}$$

If the mean density field is not known, then putting $d = 0$ in (4) at least gives the surface velocity. Finally, remembering that v is the component of horizontal velocity normal to (and to the left of) the sub-orbital track, the full vector velocity is determined only at the crossing points of such tracks, and then only if they do not intersect at too acute an angle.

(*b*) *Tides*

The tides are not geostrophic, because they are governed by both geostrophic and inertial forces of comparable magnitude. However, the variations in time of the topography, $\zeta(\theta, \lambda)$ itself, or more appropriately of its spatial gradients, contain practically all the required dynamic information if properly analysed. Vertical structure of the tidal current is unobtainable, because baroclinic motions do not deform the surface to any measurable extent; only the zero-order baroclinic mode with current uniform in depth is relevant, and this is derivable from the spatial gradients of the tidal topography. For optimum extraction of the tidal signal, it is desirable to minimize the noise caused by atmospheric loading by combining the topography from (1) with the surface pressure p_0, obtaining the 'sub-surface pressure' at the geoid:

$$g\rho(0)\zeta' = g\rho(0)\zeta + p_0. \tag{5}$$

I shall refer to $\zeta'(\theta, \lambda)$ as defined by (5) as the 'corrected topography'.

The initial object of tidal analysis of the corrected topography is to evaluate the spatial distribution of the constant admittance functions.

$$Z_2^m(\omega;\, \theta, \lambda), \quad m = 0, 1, 2,$$

of the ocean tide to the spherical harmonics of the tide-generating potential of order (species) m and degree 2 with respect to frequency ω. Alternatively, it is convenient to think of the decomposition of $\zeta'(\theta, \lambda)$ into time-harmonics of frequency ω_i and amplitude

$$H_i(\theta, \lambda) = |Z_2^m(\omega_i;\, \theta, \lambda)|\,\Gamma_i$$

with phase-lag

$$G_i(\theta, \lambda) = m\pi - \arg\{Z_2^m(\omega_i;\, \theta, \lambda)\},$$

where Γ_i are the amplitudes of discrete harmonics of the luni-solar tide-generating potential, tabulated by Cartwright & Edden (1973). (In practice, of course, H_i and G_i have to be regarded as slowly variable quantities depending on the position of the Moon's node). Possible methods of mapping Z or the discrete harmonics (H_i, G_i) will be considered later. For the present there are two further fundamental points to be noted.

Firstly, the tidal parts of $\zeta'(\theta, \lambda)$ are strictly components of the *geocentric* tide, which differs from the usual definition of the ocean tide by the addition of the vertical component of the Earth tide. The latter is supposed to be calculable by inverting an integral formula for the

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

loading effect of the relative ocean tide, but there are possible differences from the theoretical formulae (for example, a loading phase lag) that make it worth while to evaluate the Earth tide directly by analysis of the corrected topography and comparison with conventional tide measurements.

The second noteworthy point is that, because in tidal analysis we are only concerned with the temporal variations of ζ', its spatial variations may in principle be eliminated as an arbitrary constant part. This constant part may also include unremoved parts of $N(\theta, \lambda)$, which enters ζ' through (1), and so tidal analysis does not depend so critically on precise knowledge of the form of the geoid as does the estimation of the mean circulation. However, use of this property depends on frequent repetitions of the same sub-orbital track. Similarly, errors in the atmospheric loading p_0, though undesirable, will in the long run be eliminated by their lack of correlation with the tide-generating potential (ignoring a minute and correctable effect due to the solar atmospheric tide).

3. Limitations in precision

Both circulation and tidal studies require altimetry to measure ζ' continuously to an accuracy of about 0.1 m. More precisely, because the spatial gradients are the most relevant aspect of the topography, grad(ζ') should be measurable to about 0.1 m in 1000 km for gyre-scale circulation and to 0.03 m in 100 km for mesoscale eddies. Both scales are equivalent to currents at mid-latitude of order 0.003 m s^{-1}, from the first term of (4). Lower accuracy is of course tolerable for grosser features of the circulation and for tides of greater slope-amplitude.

From (1) and (5) it is clear that any error in specifying S, N, R or p_0 will affect the precision of the corrected topography ζ' or its gradients, on which all dynamical calculations depend. Errors in hydrographic data are not so serious because the surface current and tides are independent of ρ, but there are limitations in regions like the southeast Pacific, where the density field is less well known, or in regions of rapid thermocline variation. The errors to which S, N, R and p_0 are subject are different in their magnitudes and origin; they are reviewed individually below.

Altimeter height, S. Instrumentally, altimeter technology was shown by the Seasat calibration experiment (Kolenkiewicz & Martin 1982) to be precise to a few centimetres, but several corrections have to be made to convert the pulse delay to a true path length through some 800 km of space and atmosphere. There is a small correction, due to the electron density in the ionosphere, that can be supplied adequately by operating at two frequencies, and another expressible in terms of p_0 itself. The latter is no worse than the correction for atmospheric loading in (4) or (5), and this may be obtained to sufficient accuracy over the more important oceans from meteorological agencies. (Ideally, the spacecraft should carry a pressure-sounding instrument.) Water vapour in the atmosphere accounts for a substantial variable delay in travel time, and must be measured directly from the spacecraft by radiometer. Sea waves introduce a bias that is not yet entirely understood, but the wave-height measuring function of the altimeter (Tucker, this symposium) enables one to avoid at least severe wave states.

Geoid height, N. The geoid topography is of course the largest of the geophysical signals in altimetry, with spatial variations up to 100 m amplitude. It is known to about 0.1 m precision in a few sea areas such as the North Sea and part of the northwest Atlantic from dense gravity measurements. Elsewhere, the best geoid models derived from satellite orbit perturbations (e.g.

GEM-L2 from Lageos; see also Lerch *et al.* (1981)) are claimed to be correct to better than 0.1 m in their longest-wavelength components (over 6000 km). Short wavelengths, on the other hand, say less than 1000 km, can also be accurately determined by averaging the altimetry itself over many passes of past satellites, on the assumption that such steep variations in the ocean circulation are quasi-random. There is, however, a worrying range of medium-scale geoid variations for which present knowledge is inadequate. Possible dedicated satellite–satellite tracking missions such as Gravsat, dedicated to precise geopotential determination, may materialize. Otherwise, some classes of ocean monitoring by altimetry can only proceed by inference. Wunsch (1981) has proposed a model of the mean dynamic topography in the North Atlantic, derived from hydrography by inverse methods, which when subtracted from the mean altimetric geoid would provide a tolerable 'interim' gravitational geoid as a basis for dynamic monitoring. The same method could in principle be extended to other ocean areas where sufficient hydrography is known.

Orbit altitude, *R*. This is potentially the largest of all sources of error; it depends a great deal on the nature and global distribution of the systems used to track the satellite. Laser systems are very precise but are restricted to clear skies. Doppler systems (e.g. Tranet) give only 1–2 m accuracy but they have usually been better distributed and operate in all sky conditions. (Tranet-2 is a new-generation doppler system planned to give an accuracy of about 0.1 m.) However, even with optimum use of islands for tracking stations, large areas of oceanic orbit must remain untracked, and here the orbit altitude must be computed from the gravitational field of the Earth with non-trivial allowances for atmospheric drag and solar radiation. In fact the gravity field at 800 km altitude is not known well enough for the 0.1 m precision, but specially tailored approximations may be derived to suit an individual orbit, achieving altitudes precise to about 0.7 m (Lerch *et al.* 1982). A continuous 'Global Positioning System', (G.P.S.), based on satellite–satellite and ground–satellite tracking, is said to be insufficiently precise for this work at present.

If we accept the limitations of tracking and gravity models, much empirical refinement of the altitude can be achieved by using the property that the radial error is dominated by a once-per-revolution harmonic with a few much smaller harmonics due to errors in the gravity model (Lerch *et al.* 1982). Over a limited area this can alternatively be simulated by a 'bias-and-tilt' correction, adjusted to minimize the differences in the tidally corrected altimetry at the network of points where ascending and descending orbits cross. Marsh *et al.* (1982) used the last method to determine precise mean ocean topography over a considerable area of the northeast Pacific, with residual r.m.s. 'errors' in the range 0.05–0.20 m (some of which could be due to real surface motion). There is, however, a danger of removing a genuine tilt in the sea surface due to a steady circulation.

Finally, there is a limitation to resolution of the tidal signal in the topography, and hence in its removal from the circulation signal, due to synchronism with solar tides. It is understood that for the present purposes the orbit must be adjusted so that its Earth-track is repeated every *P* days, where *P* is typically in the range 3–10. Since the satellite's node regresses very slowly for the high orbital inclinations needed to cover a large range of latitude, this condition makes for a period of repeated crossing of a given Earth point close to an integral number of sidereal days, thus sampling the sidereal tides K_1 and K_2 at a very long or infinite period. Most commonly, the inclination is forced to be slightly retrograde in order to make the orbit 'Sun synchronous', for special reasons of power conservation. Sun synchronism shifts the 'frozen tide' from K_2 to S_2, which is a larger tidal constituent, while K_1 and K_2 are aliased into $1/y$ and $2/y$ frequencies respectively, to be confused with seasonal terms of interest in the circulation. There are similar problems in resolving the diurnal from the semidiurnal tides in a Sun-synchronous orbit. How to deal most effectively with this problem is outside the scope of this review, but synchronous tides may be avoided by lowering the limiting latitude to 60–70°, thus increasing the rate of nodal regression. Such an orbit is planned for the TOPEX mission, which is dedicated to the altimetric monitoring of ocean circulation (TOPEX Group 1981).

4. Results from recent missions

The only spacecraft to carry altimeters have so far been Skylab, Geos-3 and Seasat. Skylab was a pioneering experiment; although it gave encouraging profiles of the geoid to 5 m accuracy, its tracking data were too coarse to enable detection of any dynamic signals in the ocean surface. Geos-3 was more precise, both instrumentally and in its tracking support, and its large volume of data recorded in some sea areas has enabled accurate mean altimetric geoids to be constructed by minimizing crossing-point differences (Marsh *et al.* 1980).

The combined circumstances, of orbital error being confined to large wavelengths and excellent geoidal definition in the northwest Atlantic, enabled Leitao *et al.* (1979) to give the first demonstration of the 1 m steep rise of ζ across the Gulf Stream and a 0.5 m hump corresponding to a warm eddy to its north. These and other diagnoses were confirmed by AXBT data and infrared imagery from N.O.A.A. satellites. Much work in the same area was done with greater precision from the Seasat altimetry, notably by Cheney & March (1981), whose diagram of eight repeated passes over a slowly changing cold ring south of the Gulf Stream is now famous.

Cheney & Marsh (personal communication) have also experimented with smoothing the mean topography derived from global Seasat altimetry and the GEM-L2 geoid to obtain a circulation pattern that may be considered accurate at wavelengths greater than about 6000 km. Bruce Douglas of N.O.A.A. has done the same type of experiment over a 3 day arc. Draft copies of both maps appear to reproduce the major known gyre patterns, with some anomalies.

As well as its greater overall precision, the short-lived career of Seasat included an important period of 25 days when its orbit was arranged to repeat its Earth-track precisely with a period of 3 days 13 min. The results from the repeat orbits showed that this condition is essential if one is to distinguish temporal variations in ζ from spatial variations in N in areas where the geoid N is not accurately known. The most outstanding results of this sort are the mapping of the mesoscale variability of the ocean from the variation of grad(ζ') about its 25 day mean value at each point. (This measure can be roughly converted into kinetic energy at crossing-points.) The procedure was apparently first suggested and carried out by Menard (1982) for the areas bordering the Gulf Stream and the Kuroshio. His map of the latter region is reproduced in figure 1. A similar map covering the world's oceans has been produced by Cheney *et al.* (1982). As well as a surprisingly large variability in the vicinity of the Agulhas and Falkland currents and over much of the Southern Ocean, their map also shows interesting large zones of quiescence in the eastern parts of the Atlantic and Pacific oceans.

All the investigations mentioned above were preceded by removal of the tidal elevation from ζ' by means of one of the computer models mentioned in §1, and paid no further attention to any possible residual tidal signal. A few researchers, however, have examined the tidal content

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

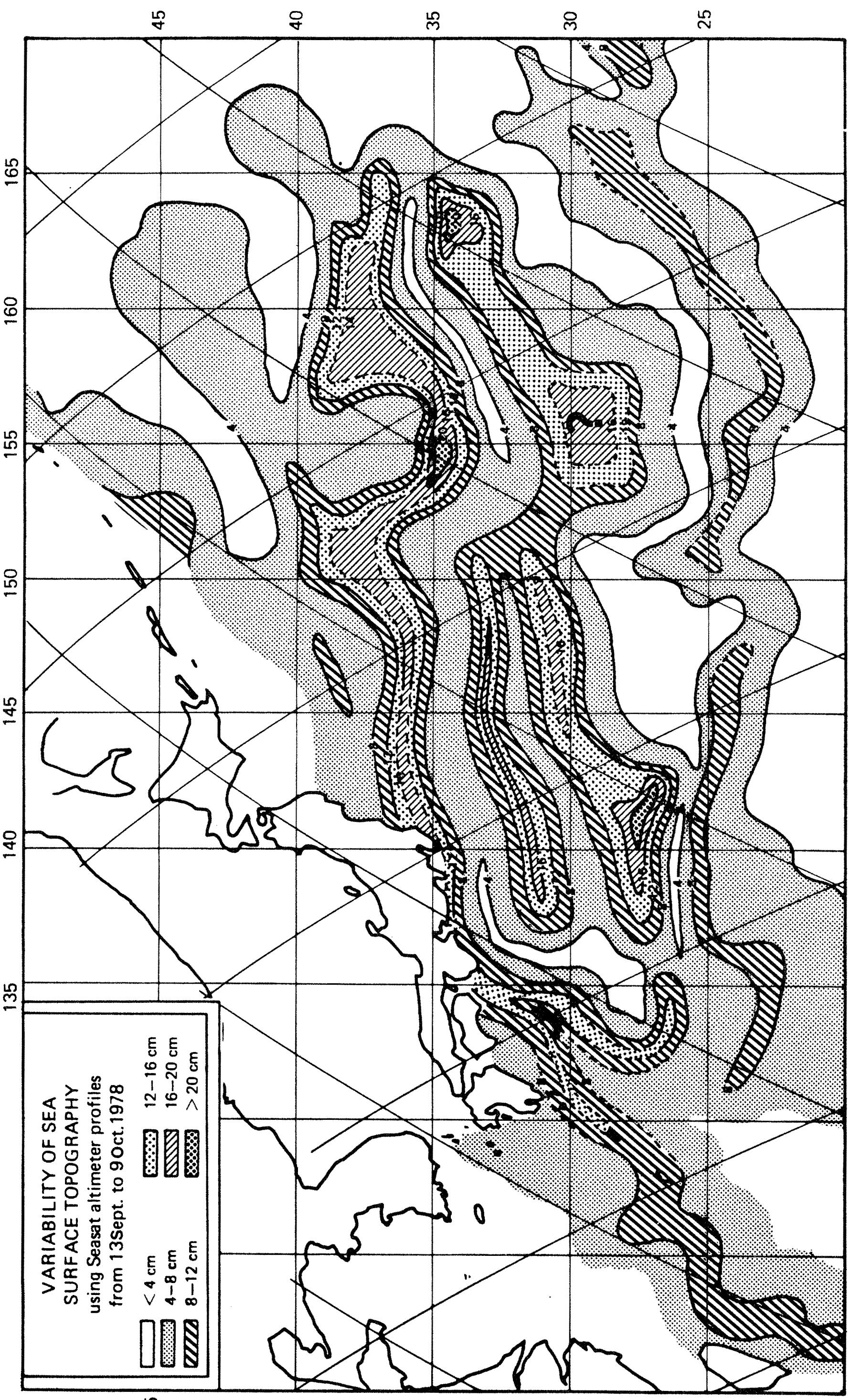

FIGURE 1. Variability of sea level in the northwest Pacific, derived from Seasat altimetry along the shown orbital tracks repeated every 3 days during 13 September to 9 October 1978. Shaded areas denote intervals of r.m.s. variation from less than 4 cm (white) to more than 20 cm. (From Menard (1982), by permission.)

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

of raw values of ζ' or of its gradients. Le Provost & Brossier (1981) and Parke (1981) showed clear evidence of tidally varying slopes in the topography over shallow shelf waters, where the signal is strengthened by the compressed horizontal scale of movement, and in any case the oceanic computer models are inadequate. Both these authors relied again on the repeated orbit period of Seasat.

Diamante & Nee (1981) fitted a mean sea surface and five tidal harmonics to the (non-repeating) Geos-3 data over the Bermuda calibration area, after subtracting a fairly good geoid estimate and adjusting orbital biases and tilts in an iterative solution. Their results compare reasonably well with deep-sea tidal measurements in the area, unlike those of other authors who appear to have been unable to obtain adequate tidal constants from Geos data.

Again working with the 25 days repeated orbit of Seasat, Cartwright & Alcock (1981) devised a method of tidally analysing the spatial differences in S between all adjacent crossing points in a network, without geoidal correction, but after corrections for p_0 and the Body Tide of the solid Earth. All harmonic constituents were lumped together into a three-parameter tidal solution for each S-difference, these differences being constrained to sum to zero round each mesh element of the orbital grid. Results for the M_2 constituent over a sizable area of the northeast Atlantic agreed well with a tidal map based on an extensive network of direct measurements, surprisingly for such a short span of satellite data. The same authors (Cartwright & Alcock 1983) show similar results for an even larger area of the northeast Atlantic (figure 2), and a less successful attempt to map the tides in the noisy area east of Newfoundland.

Using the same data set, Mazzega (1982) has fitted a set of complex spherical harmonics up to order (3, 3) to the M_2 tide in the Indian Ocean, with an approximate averaging assumption for partial removal of other tidal harmonics. His resulting tidal map is surprisingly realistic. Here, as in the work of Cartwright & Alcock, simplifying assumptions necessary to produce a plausible result from the very short span of Seasat's repeated orbit could be removed or refined if applied to a year or more of good data.

5. Summary and future prospect

There is no doubt from the above results that precision in satellite altimetry and in its associated technology has passed the elementary stage of rough geoid determination and has reached a point where information about major current features and tides can be extracted. The performance of Seasat is generally admitted to have surpassed all expectations. Nevertheless, the precisions of its orbit R and of the known geoid N were not sufficient for even a brief determination of ocean-wide circulation, except perhaps at extremely long wavelengths. Orbital precision for Seasat was limited by its relatively low altitude of 800 km and by the bulky shape of its antennae, designed to perform 'synthetic aperture radar' imagery, thus introducing indeterminate drag forces and solar radiation pressures.

The precision in all aspects of altimetry required for a serious monitoring of ocean circulation (perhaps somewhat less precision is needed for tidal studies), is really very demanding of modern space technology. It requires something a little better than even 'Seasat', and this can only be achieved by a satellite system completely dedicated to this one objective. It must have a greater altitude to avoid atmospheric drag, a reasonably symmetrical shape, a globally distributed set of tracking stations, a radiometer for water vapour assessment, a scatterometer for wind-stress, and preferably a sounder for surface pressure. Its orbit must produce a repeated

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

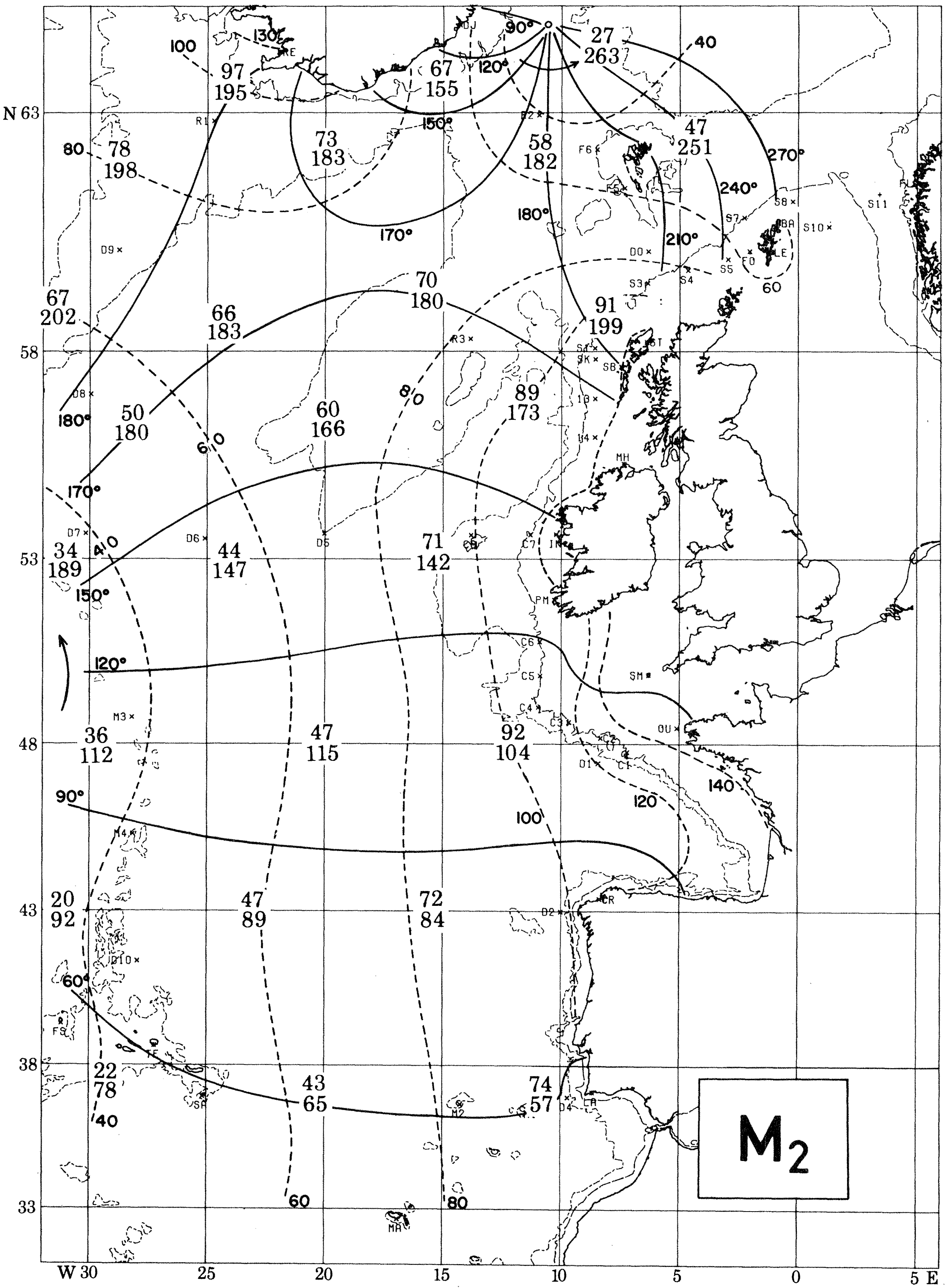

FIGURE 2. Tidal constants for M_2 in the northeast Atlantic from bottom pressure measurements (contours) and from analysis of differences in Seasat altimetry between orbital crossing points. Pairs of large figures give amplitude in centimetres (upper) and phase lag in degrees (lower) at crossing points, resulting from altimetric analysis. (From Cartwright & Alcock (1983).)

Earth track with a period of order 6–10 days (for sufficiently dense spatial coverage) and respectable crossing angles, with an inclination chosen to avoid Sun-synchronism and similar tidal synchronisms. Of the three or four altimetric satellites being seriously projected at the present time for launch before the end of the 1980s, only TOPEX (TOPEX Group 1981) meets most of these requirements. The rest have other declared objectives and so will fall short of the ideal set of requirements for climatic study of ocean circulation. At best, they will provide extended monitoring of the Seasat calibre, with special interest in seasonal variations of mesoscale energy and better resolution of lunar tides.

Besides the above requirements, absolute results for the mean circulation need a gravitational geoid N, precise to 0.1 m, to derive the dynamic element of the topography ζ from (1). Such a geoid may come sooner or later through a dedicated mission such as Gravsat, but this in itself requires a great deal of finance and scientific effort. In my view, plans for synoptic monitoring of the ocean circulation should not rely on such a bonus. A more realistic approach is to adopt Wunsch's (1981) suggestion, by applying a mean topographic surface derived from hydrographic measurements to the mean altimetric surface, if necessary for a concentrated effort on the Atlantic Ocean alone, and to use the resulting topography as an interim substitute reference-surface until a better geoid is derived by an independent method. It should not be forgotten that ship-based measurements will always be required as an essential adjunct to remote sensing of the ocean surface.

References

Cartwright, D. E. & Alcock, G. A. 1981 In *Oceanography from space* (ed. J. F. R. Gower), pp. 885–895. New York: Plenum.

Cartwright, D. E. & Alcock, G. A. 1983 In *Seasat over Europe* (ed. T. D. Allan). (In the press.) Chichester: Ellis Horwood.

Cartwright, D. E. & Edden, A. C. 1973 *Geophys. Jl R. astr. Soc.* **33**, 253–264.

Cheney, R. E. & Marsh, J. G. 1981 *J. geophys. Res.* **86** (C1), 473–483.

Cheney, R. E., Marsh, J. G. & Beckley, B. D. 1982 *J. geophys. Res.* (In the press.)

Diamante, J. M. & Nee, T.-S. 1981 In *Oceanography from space* (ed. J. F. R. Gower), pp. 907–918. New York: Plenum.

Kolenkiewicz, R. & Martin, C. F. 1982 *J. geophys. Res.* **87** (C5), 3189–3197.

Leitao, C. D. & Huang, N. E. 1979 *J. geophys. Res.* **84** (B8), 3969–3973.

Le Provost, C. & Brossier, C. 1981 In *Oceanography from space* (ed. J. F. R. Gower), pp. 927–932. New York: Plenum.

Lerch, F. J., Putney, B. H., Wagner, C. A. & Klosko, S. M. 1981 *Mar. Geod.* **5**, 145–187.

Lerch, F. J., Marsh, J. G., Klosko, S. M. & Williamson, R. G. 1982 *J. geophys. Res.* **87** (C5), 3281–3296.

Marsh, J. G., Cheney, R. E., Martin, T. V. & McCarthy, J. J. 1982 *Eos, Wash.* **63**, 178–179.

Marsh, J. G., Martin, T. V., McCarthy, J. J. & Chovitz, P. S. 1980 *Mar. Geod.* **3**, 359–378.

Mazzega, P. 1982 The M2 oceanic tide recovered from Seasat altimetry in the Indian Ocean. (Unpublished.) C.N.E.S./G.R.G.S., Toulouse.

Munk, W. & Wunsch, C. 1982 *Phil. Trans. R. Soc. Lond.* A **307**, 439–464.

Parke, M. E. 1981 In *Oceanography from space* (ed. J. F. R. Gower), pp. 919–925. New York: Plenum.

Parke, M. E. 1982 *Mar. Geod.* **6**, 35–81.

Parke, M. E. & Hendershott, M. C. 1980 *Mar. Geod.* **3**, 379–408.

Schwiderski, E. W. 1980 *Mar. Geod.* **3**, 161–255.

TOPEX Group 1981 *Satellite altimetric measurements of the ocean.* Pasadena: Jet Propulsion Laboratory.

Wunsch, C. 1978 *Rev. Geophys. Space Phys.* **16**, 583–620.

Wunsch, C. 1981 *Mar. Geod.* **5**, 103–119.

Wunsch, C. & Gaposchkin, E. M. 1980 *Rev. Geophys. Space Phys.* **18**, 725–745.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES